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Frequency-dependent magnetotransport in a two-dimensional magnetic modulation system

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Abstract

We analyze the dynamics of a charged particle moving in the presence of spatially modulated magnetic fields, motivated by recent transport experiments by Ye et al. (Phys. Rev. Lett. 74 (1995) 3013). We show from Poincaré surfaces of section that the ratio of pinned orbits to chaotic orbits depends strongly on the energy and the structure parameters. We present a complete characterization of the dynamical behavior of such structures, and calculate the magnetoconductivity using a classical Kubo formula. We investigate the contribution to the conductivity from pinned and runaway orbits. Although the DC conductivity of the system depends strongly on the ratio of pinned to runaway trajectories, the high-frequency response reflects the topology of the different orbits. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Commensurability oscillations in two-dimensional electron gases (2DEG) have attracted much attention recently. These oscillations arise when the 2DEG is exposed to a uniform magnetic field and an imposed superstructure (electrostatic or magnetic), set of potential barriers. These oscillations result from two length scales: the cyclotron radius $R_c = v/\omega_o$, where $\omega_o = eB_o/mc$, and the period of the superstructure a . The case where the potential barriers are defined by electrostatic modulation has been studied intensively and the low field oscil-

lations in the magnetoresistance have been observed [1–4]. For low and moderate fields, and in high mobility samples (Fermi wavelength $\lambda_F \ll R_c$) Landau level quantization can be neglected and the classical approach is indeed sufficient to describe the dynamics and the magnetotransport [5,6].

In this paper we investigate a model where the second length scale in the problem, apart from the cyclotron radius R_c , is defined through a periodic variation in the magnetic field itself, instead of an additional electrostatic modulation [7]. We investigate magnetotransport in a model assuming a smooth and infinite antidot potential that can be written as

$$\mathbf{B} = \hat{z}B_o \left[1 + \frac{r}{2} \left(\cos \frac{2\pi x}{a} + \cos \frac{2\pi y}{a} \right) \right], \quad (1)$$

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where r is the field modulation strength, $r = B_m/B_o$. We show that chaos and quasiperiodic orbits are clearly reflected in the frequency-dependent magnetotransport in the lateral surface of the superstructure. Similar behavior has been reported by Vasiliadou et al. in an “electrostatic” antidot array [8].

2. Electron dynamics

Assume a 2DEG in the xy plane with a perpendicular magnetic field given by Eq. (1) where $r < 1$. The system can be described by the Hamiltonian $H = [\mathbf{p} + (e/c)\mathbf{A}(\mathbf{r})]^2/2m$, where \mathbf{A} is the vector potential, and m is the effective mass in GaAs, and we choose a symmetric gauge

$$\mathbf{A} = \left(-B_o \frac{y}{2} - \frac{B_m}{2k} \sin ky, B_o \frac{x}{2} + \frac{B_m}{2k} \sin kx, 0 \right), \quad (2)$$

where $k = 2\pi/a$. For $r \neq 0$ the Hamiltonian leads to a nonlinear coupling of both degrees of freedom and no analytical solution can be given in general. We characterize the type of a trajectory in phase space (x, \dot{x}, y, \dot{y}) by means of Poincaré surfaces of section on the xy -plane at \dot{x} maximum and $\dot{y} = 0$. Due to the translational symmetry of the superstructure potential in the xy plane, we fold the Poincaré section to one unit cell xy in the range $(-\frac{1}{2}, \frac{1}{2})$. Thus, the phase-space trajectories are uniquely determined by points in the surface of section.

For a given modulation strength r , the ratio between pinned and chaotic orbits depends on $2R_c/a$ and not on the energy and the uniform field individually. In Fig. 1 we show two Poincaré sections: Fig. 1a $2R_c/a = 1.4$ and $r = 0.6$, Fig. 1b $2R_c/a = 0.4$ and $r = 0.75$. Even though the strength of the modulation in Fig. 1b is higher than in Fig. 1a the chaotic region in the first section is larger than in the second. This can be understood from the fact that the cyclotron radius in the first case is 7 times larger than the second one, so that the electron is more likely to “collide” with many maxima in the modulation. In Fig. 1b we find more quasiperiodic regions than in Fig. 1a since in Fig. 1b R_c is smaller than the superstructure length

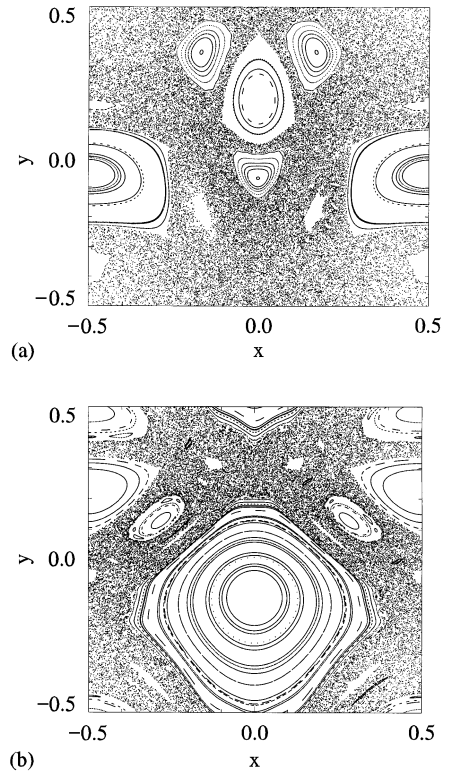


Fig. 1. (a) and (b) Poincaré surface sections for the same energy, but different $2R_c/a$ and r .

scale a , so that it is easier to form pinned orbits with different topology, giving rise to different resonances, than when R_c is of the order of a . We will see on the AC-conductivity some resonances that reflect cyclotron-like orbits with $\omega \approx \omega_c(1 \pm r)$ in the case related to Fig. 1b, while not for Fig. 1a.

3. Magnetotransport

A quantitative theory of magnetotransport requires detailed consideration of regular (pinned) and chaotic (runaway) orbits in the presence of impurity scattering. In Hamiltonian systems where the volume in phase space (x, p_x, y, p_y) is conserved, the conductivity tensor σ is the sum of the individual contribution of trajectories weighted by their volume in phase space. We will show numerically that the contribution of pinned orbits to the

DC-conductivity is very small, while they are the ones related to the peaks in the AC-conductivity. In the AC case where the B_0 field, r , and a are constant, we will assume that the proportion of pinned to runaway orbits does not change even after considering impurity scattering (which should be the case if mobility in the unmodulated system is high).

We calculate the conductivity by using a classical linear response expression (Kubo formula) [9]

$$\sigma_{ij}(\omega) = \frac{ne^2}{k_B T} \int_0^\infty dt e^{-t/\tau} e^{i\omega t} \langle V_i(t) V_j(0) \rangle, \quad (3)$$

where $\langle V_i(t) V_j(0) \rangle = \int dx_o dy_o \int dv_{x_o} dv_{y_o} V_i(t) V_j(0)$ is the velocity correlation function averaged over initial conditions in phase space (to substitute for an ensemble average), n is the 2D electron density, k_B the Boltzmann constant, T the temperature, and τ is a phenomenological scattering time associated with the remnant random impurity and alloy scattering in the real system. We choose a typical experimental value for the scattering time, $\tau \approx 3 \times 10^{-11}$ s, and evaluate the magnetoresistance as

$$\rho_{xx} = \sigma_{xx} / (\sigma_{xx}^2 + \sigma_{xy}^2), \quad (4)$$

since $\sigma_{xx} = \sigma_{yy}$ and $\sigma_{xy} = \sigma_{yx}$ due to the symmetry of the superstructure.

We calculate the velocity correlation function $\langle V_i(t) V_j(0) \rangle$ by generating random sets of initial conditions in (x, p_x, y, p_y) keeping the energy constant.

We have calculated the DC-resistivity for the same parameters used in the experiment of Ye et al. In Fig. 2 we show ρ_{xx} , where $2R_c/a = 0.57/B$. The amplitude of ρ_{xx} oscillates in $1/B$ and it has minima at values $2R_c/a \approx 1.22, 2.20, 3.17$. These oscilla-

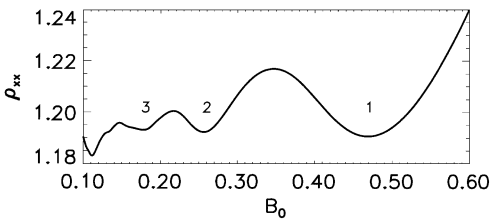


Fig. 2. DC-resistance ρ_{xx} versus the uniform magnetic field B_0 the positions of the $1/B$ oscillations are shown.

tions have been predicted by Peeters et al. to be at $2R_c/a = n + \frac{1}{4}$ [7]. Our values, as well as the experimental values, seem to be close to the predicted ones. These oscillations are closely related to the magnetoresistance oscillations found in a weak electrostatic modulation but with the characteristic difference between magnetic and electric oscillations appearing in their phase [1].

For the calculations shown in Fig. 3a and b we also used random sets, but classified the orbits by

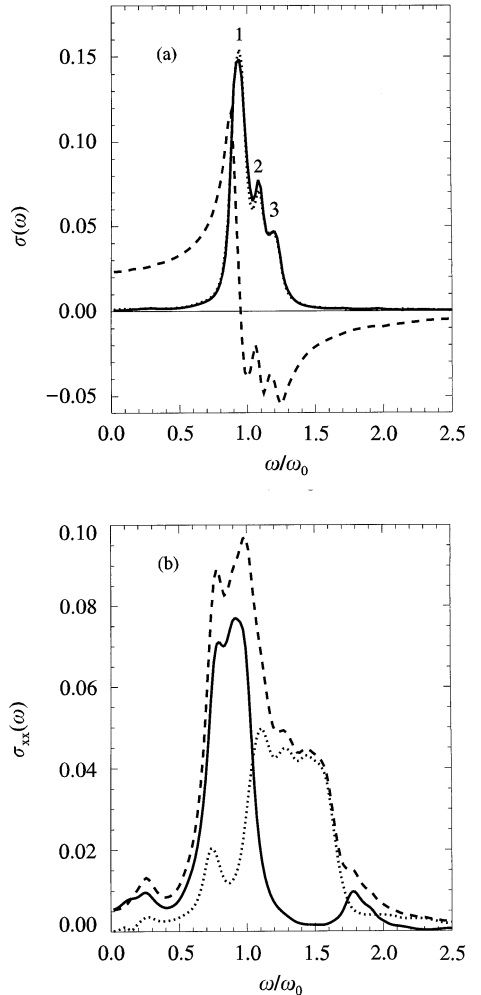


Fig. 3. (a) The solid line shows σ_{xx} , the dotted line shows σ_{yy} and the dashed line shows σ_{xy} . (b) All the lines show σ_{xx} . The solid line is the contribution due to chaotic orbits, the dotted line is the contribution due to pinned orbits and the dashed line is the total.

inspecting the corresponding Poincaré sections, so that we were able to separate the initial conditions that were related to pinned or chaotic orbits. The AC-conductivity when the modulation strength r is zero yields the classical Drude peak with half-width $1/\tau$. Once the modulation is turned on, however, and for small r , the Drude peak remains centered around $\omega/\omega_c = 1$, but the conductivity peak appears to have some inhomogeneous broadening. This broadening appears *before* the chaotic orbits appear, i.e., the Poincaré section shows only quasi-periodic orbits, and it is associated with fact that the electron trajectories still have the characteristic frequency ω_c , but due to the small modulation the electron orbits precess with other characteristic frequencies. By increasing the modulation strength some features will appear in $\sigma(\omega)$ at the same time that chaos and Kolmogorov–Arnol’d–Moser (KAM) islands appear in the Poincaré surface section. Due to the chaotic orbits an offset at zero frequency appears, and by increasing the portion of chaotic trajectories the offset increases at the expense of the resonance peaks.

In Fig. 3a there are three resonances in $\sigma_{xx}(\omega)$ and $\sigma_{yy}(\omega)$. The first and third peaks are related to the pinned orbits while the second is a contribution from the runaway orbits. The traces are for the same parameters used in the Poincaré section in Fig. 1a and since the pinned orbits show two major peaks, they appear related to each of the quasi-periodic regions in Fig. 1a. In Fig. 3b there are two features at $\omega/\omega_c \approx (1 \pm 0.75)$. These two features are more pronounced in the chaotic contribution (solid trace) than in the pinned one (dotted trace). At $\omega/\omega_c = 0$, the pinned orbits trace show a zero, indicating that the chaotic orbits are more impor-

tant for the DC-transport, as expected. Finally, comparing Fig. 3a and b, we see more features in the latter, clear contribution from the more frequent pinned orbits.

We have studied the frequency-dependent magnetotransport in 2D magnetic field modulation in a square lattice. Our DC results reflect those seen experimentally and predicted before. This study has opened a new class of resonances in the AC-conductivity, which would reflect the different character of the various electron trajectories, and the degree of integrability (or non-) of these systems. The study of different profiles of magnetic field modulation and even different lattice structures, both theoretically and experimentally, should give us a better insight into the microscopic character of the electron trajectories in different regimes.

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