Resonant Scattering in a Strong Magnetic Field: Exact Density of States

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We study the structure of 2D electronic states in a strong magnetic field in the presence of a large number of resonant scatterers. For an electron in the lowest Landau level, we derive the *exact* density of states by mapping the problem onto a zero-dimensional field-theoretical model. We demonstrate that the interplay between resonant and nonresonant scattering leads to a *nonanalytic* energy dependence of the electron Green function. In particular, for strong resonant scattering the density of states develops a *gap* in a finite energy interval. The shape of the Landau level is shown to be very sensitive to the distribution of resonant scatterers. [S0031-9007(97)04376-7]

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During recent years there has been a growing interest in the role of multiple resonant scattering in transport. Most of the studies have been related to the passage of light through a disordered medium. In particular, it was shown in a recent experiment [1] and subsequent works [2] that multiple scattering near resonances leads to a renormalization of the diffusion coefficient up to an order of magnitude.

It is natural to expect that resonant scattering would also affect quite strongly the properties of electrons in disordered systems. The effective trapping of the electron in resonant states is expected to suppress diffusion, just as in optics [1,2]. This would in turn be evident in the single-particle density of states (DOS) and localization properties.

In this paper we study the electronic states of a 2D system in a strong magnetic field in the presence of a *large* number of resonant scatterers. This choice is motivated in part because experimental structures with such geometry recently became available thanks to remarkable advances in the fabrication of arrays of ultrasmall self-assembled quantum dots [3]. With typical sizes of less than 20 nm and very narrow variations of less than 10%, an array of such dots with density $10^{10}-10^{11}$ cm^{-2} can be produced at some preset distance from a plane of a high mobility electron gas [4]. As the Fermi energy in the plane approaches the levels of dots, the virtual transitions between dots and the plane result in multiple resonant scattering. Such scattering which, in principle, extends through the entire system, strongly affects the DOS of a 2D electron.

The DOS of 2D disordered electronic systems in a quantizing magnetic field has been extensively studied for the past two decades [5–15]. The macroscopic degeneracy of the Landau levels (LL) makes a perturbative treatment of even weak disorder impossible and calls for nonperturbative approaches. For high LL, Ando and Uemura's self-consistent Born approximation [5] was shown to be asymptotically exact for short-range disorder [11,13], while in the case of long-range disorder the DOS can be obtained within the eikonal approximation [13]. For low LL and uncorrelated disorder, the problem contains no small parameter and neither of those approximations apply. Nevertheless, Wegner was able to obtain the exact DOS in a white-noise potential for the lowest LL, by mapping the problem onto that of the 0D complex ϕ^4 model [9]. This remarkable result was extended to non-Gaussian random potentials by Brezin *et al.* [10], and recently to multilayer systems [15].

The "regular" disorder broadens the LL into a band of width Γ. At the same time, the resonant scattering leads to a sharp energy dependence of the DOS near the resonance. The scattering is enhanced close to the LL center and is suppressed in the tails. Therefore, the *efficiency* of resonant scattering is characterized by the ratio γ/Γ, where γ is the width of the energy spread of resonant states.

The interplay of the resonant and nonresonant scattering leads to a rather complex energy dependence of the DOS. Nevertheless, for the lowest LL the problem can be solved *exactly* [see Eqs. (8) and (9) below]. We exploit the hidden supersymmetry of the lowest LL [9,10] in order to map the averaged Green function onto a version of 0D field theory. The DOS appears to be *nonanalytic* as a function of energy; in particular, it develops a *gap* as resonant scattering becomes strong.

The model.—Consider a 2D electron gas separated by a tunneling barrier from a system of localized states (LS). In addition to LS, a Gaussian random potential $V(\mathbf{r})$ with correlator $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = w\delta(\mathbf{r}-\mathbf{r}')$ is present in the plane. We assume that energies of LS are close to the lowest LL and adopt the tunneling Hamiltonian,

$$\hat{H} = \sum_{\mu} \epsilon_{\mu} a_{\mu}^{\dagger} a_{\mu} + \sum_{i} \epsilon_{i} c_{i}^{\dagger} c_{i} + \sum_{\mu,i} (t_{\mu i} a_{\mu}^{\dagger} c_{i} + \text{H.c.}), \tag{1}$$

where ϵ_{μ}, $c_{\mu}^{\dagger}$, and c_{μ} are the eigenenergy, creation, and annihilation operators of the eigenstate $|\mu\rangle$ of the Hamiltonian $H_0 + V(\mathbf{r})$ (H_0 describes a free electron

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in magnetic field), ϵ_i, $c_i^\dagger$, and c_i are those of the ith LS, and $t_{\mu i}$ is a tunneling matrix element. The latter is defined as $t_{\mu i} = \int d\mathbf{r}\,dz\,\psi_\mu^*(\mathbf{r},z)V_i(\mathbf{r},z)\psi_i(\mathbf{r},z) \simeq \psi_\mu^*(\mathbf{r}_i,z_i)\int d\mathbf{r}\,dz\,V_i(\mathbf{r},z)\psi_i(\mathbf{r},z)$, where $V_i(\mathbf{r},z)$ is the LS potential and $\psi_i(\mathbf{r},z)$ is its wave function. In the direction normal to the plane, the wave function $\psi_\mu^*(\mathbf{r},z)$ decays as $e^{-\kappa z}$, while in the plane it behaves as an eigenfunction $\psi_\mu^*(\mathbf{r})$ of the Hamiltonian $H_0 + V(\mathbf{r})$. For high enough tunneling barrier, the dependence of κ on μ can be neglected [16] so that $t_{\mu i} \simeq \psi_\mu^*(\mathbf{r}_i)t_i$.

A formal expression for the Green function of a 2D electron with energy ω, $G_{\mu\nu}(\omega) = \langle\mu|(\omega - \hat{H})^{-1}|\nu\rangle$, can be derived by integrating out the LS degrees of freedom. It has the form $\hat{G}(\omega) = (\omega - \hat{\epsilon} - \hat{\Sigma})^{-1}$, where $\hat{\epsilon}$ is a diagonal matrix with elements ϵ_μ, and self-energy matrix,

$$\Sigma_{\mu\nu}(\omega) = \sum_i \frac{t_{\mu i}t_{i\nu}}{\omega - \epsilon_i} = \sum_i \frac{t_i^2\psi_\mu^*(\mathbf{r}_i)\psi_\nu(\mathbf{r}_i)}{\omega - \epsilon_i}, \qquad (2)$$

comes from scattering of the electron by LS. In such a form, however, the Green function is hard to analyze. Instead, it is convenient to work with an effective *in-plane* Hamiltonian, H_{eff}, for the electron with energy ω. Recasting $\Sigma_{\mu\nu}$ in coordinate representation, we obtain $H_{\text{eff}}(\omega) = H_0 + V(\mathbf{r}) + U(\omega,\mathbf{r})$, where the last term,

$$U(\omega,\mathbf{r}) = \sum_i \frac{t_i^2}{\omega - \epsilon_i}\,\delta(\mathbf{r}_i - \mathbf{r}), \qquad (3)$$

describes the resonant scattering of electrons by the LS. The potential (3) resembles that of pointlike scatterers. The crucial difference, however, is that here scattering strength depends on the proximity of the electron energy to the LS levels. It is important to notice that $U(\omega,\mathbf{r})$ changes from repulsive to attractive as the electron energy passes through the resonance. Since positions of LS are random with uniform density n_{LS}, the distribution function of U is Poissonian.

In the following, we assume that the tunneling barrier is high enough, and neglect the difference in t_i for different LS, setting $t_i = t$ in the rest of the paper. A strong magnetic field implies that scattering retains electrons in the lowest LL. While for the white-noise potential this condition is standard, it is more restrictive for the resonant scattering. It should be noted, however, that the latter is effectively reduced by the energy spread of LS.

Calculation of the DOS, $g(\omega) = -\pi^{-1}\text{Im}\,\overline{G(\mathbf{r},\mathbf{r})}$, requires averaging of the Green function, $G(\mathbf{r},\mathbf{r}) = \langle\mathbf{r}|(\omega_+ - H_{\text{eff}})^{-1}|\mathbf{r}\rangle$ (with $\omega_+ = \omega + i0$), over *both* random potentials $V(\mathbf{r})$ and $U(\omega)$. Below, we derive this DOS exactly by using the approach of Ref. [10].

Derivation of DOS.—The Green function is presented as a bosonic functional integral $G(\mathbf{r},\mathbf{r}) = -iZ^{-1} \times \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{iS}\psi(\mathbf{r})\bar{\psi}(\mathbf{r})$ with the action $S[\bar{\psi},\psi] = \int d\mathbf{r}\,\bar{\psi}(\mathbf{r})[\omega_+ - H_{\text{eff}}(\omega)]\psi(\mathbf{r})$. After writing the normalization factor as a fermionic integral $Z^{-1} = \int \mathcal{D}\chi\mathcal{D}\bar{\chi}e^{iS}$ with the same action $S[\bar{\chi},\chi]$, both ψ and χ are projected on the lowest LL subspace as $(\omega - H_0)\psi = \omega\psi$ (measuring all energies from the lowest LL). In the symmetric gauge, this projection is achieved with $\psi = (2\pi l^2)^{-1/2}e^{-|z|^2/4l^2}u(z)$ and $\chi = (2\pi l^2)^{-1/2}e^{-|z|^2/4l^2}v(z)$, where the bosonic field $u(z)$ and the fermionic field $v(z)$ are analytic functions of the complex coordinate $z = x + iy$ (l is the magnetic length). The Green function then takes the form $G(\mathbf{r},\mathbf{r}) = -i(2\pi l^2)^{-1}e^{-|z|^2/2l^2}\langle u(z)\bar{u}(z^*)\rangle$, where $\langle\cdots\rangle$ denotes a functional integral over $u(z)$ and $v(z)$ with the action

$$S = \int \frac{d^2z}{2\pi l^2}\,e^{-|z|^2/2l^2}(\bar{u}u + \bar{v}v)[\omega_+ - V - U(\omega)]. \qquad (4)$$

As a next step, one introduces Grassman coordinates θ and θ^*, normalized as $\int d^2z\,d^2\theta\,e^{-|z|^2-\theta\theta^*} = 1$, and defines analytic "superfields" $\Phi(z,\theta) = u(z) + \theta v(z)/\sqrt{2}\,l$ and $\bar{\Phi}(z^*,\theta^*) = \bar{u}(z^*) + \theta^*\bar{v}(z^*)/\sqrt{2}\,l$, taking values in the "superspace" $\xi = (z,\theta)$. Using $\langle u\rangle = \langle v\rangle = 0$ and $\langle u\bar{u}\rangle = \langle v\bar{v}\rangle$, the Green function can be presented as

$$G = -i\,\frac{e^{-\xi\xi^*/2l^2}}{2\pi l^2}\int \mathcal{D}\Phi\mathcal{D}\bar{\Phi}e^{iS}\Phi(\xi)\bar{\Phi}(\xi^*), \qquad (5)$$

where $\xi\xi^* \equiv |z|^2 + \theta\theta^*$ and $S[\bar{\Phi},\Phi]$ is obtained from (4) by substituting $\bar{u}u + \bar{v}v = 2\pi l^2 \times \int d^2\theta\,e^{-\theta\theta^*/2l^2}\bar{\Phi}(\xi^*)\Phi(\xi)$.

We now perform the ensemble averaging over V and U. The Gaussian averaging of $\exp(-i\int VQd^2z)$, where $Q = \int d^2\theta\,e^{-\xi\xi^*/2l^2}\bar{\Phi}(\xi^*)\Phi(\xi)$, gives $\exp[-(w/2)\int Q^2d^2z]$, while the averaging of $\exp(-i\int UQd^2z)$ with a Poissonian distribution function [17] yields

$$\exp\left\{-n_{ls}\int\left[1 - \left\langle\exp\left(-\frac{it^2Q}{\omega - \epsilon}\right)\right\rangle_\epsilon\right]d^2z\right\}, \qquad (6)$$

where $\langle\cdots\rangle_\epsilon$ denotes energy averaging. As a result, one obtains the following effective action

$$\begin{aligned} iS[\Phi,\bar{\Phi}] = {} & i\omega_+\int d^2\xi\,\alpha - \frac{\Gamma^2}{2}\int\frac{d^2z}{2\pi l^2} \\ & \times\left(2\pi l^2\int d^2\theta\,\alpha\right)^2 - \nu\int\frac{d^2z}{2\pi l^2} \\ & \times\left\{1 - \left\langle\exp\left[-i\lambda 2\pi l^2\int d^2\theta\,\alpha\right]\right\rangle_\epsilon\right\}, \qquad (7)\end{aligned}$$

where $\alpha(\xi,\xi^*) = e^{-\xi\xi^*/2l^2}\bar{\Phi}(\xi^*)\Phi(\xi)$. Here $\Gamma = (w/2\pi l^2)^{1/2}$ is Wegner's width of lowest LL (in the absence of resonant scattering), $\nu = 2\pi l^2 n_{\text{LS}}$ is the "filling factor" of LS, and we denoted $\lambda = \delta^2/(\omega - \epsilon)$, where $\delta = t/(2\pi l^2)^{1/2}$ characterizes the tunneling.

The action (7) possesses a supersymmetry, characteristic for the lowest LL [9,10]. Being evident for the first term, this symmetry between z and θ can be made explicit for the second and third terms also by making use of the identity [10] $n(2\pi l^2\int d^2\theta\,e^{-\theta\theta^*/2l^2}\bar{\Phi}\Phi)^n = 2\pi l^2\int d^2\theta\,e^{-n\theta\theta^*/2l^2}(\bar{\Phi}\Phi)^n$, which allows one to replace any functional of the form $\int d^2z\,f(2\pi l^2\int d^2\theta\,\alpha)$ with

$2\pi l^2 \int d^2\xi h(\alpha)$, where $\partial h(x)/\partial x = f(x)/x$. As a result, one obtains a manifestly supersymmetric action $S = \int d^2\xi \mathcal{A}(\alpha)$, where

$$i\mathcal{A}(\alpha) = i\omega_+\alpha - \frac{\Gamma^2\alpha^2}{4} - \nu \int_0^\alpha \frac{d\beta}{\beta}\left[1 - \left\langle \exp\left(-\frac{i\delta^2\beta}{\omega - \epsilon}\right)\right\rangle_\epsilon\right]. \quad (8)$$

The supersymmetry leads, in turn, to the exact cancellation of contributions from z and θ spatial integrals into each diagram, so that the entire perturbation series can be generated in the 0D field theory with the *same* action [9,10]. The Green function is then given by the ratio of two ordinary integrals, $G(\omega) = -i(2\pi l^2)^{-1}Z_0^{-1}\int d^2\phi e^{i\mathcal{A}}\phi\phi^*$, where $Z_0 = \int d^2\phi e^{i\mathcal{A}}$ with $\mathcal{A}(\phi\phi^*)$ from (8). From this Green function, the DOS is obtained as

$$g(\omega) = \frac{1}{2\pi^2 l^2}\,\mathrm{Im}\,\frac{\partial}{\partial\omega_+}\,\ln\int_0^\infty d\alpha\, e^{i\mathcal{A}(\alpha)}, \quad (9)$$

where the derivative applies only to the first term of (8).

Examples.—The energy averaging in (8) can be performed analytically for an arbitrary distribution of LS levels, $f_\gamma(\epsilon - \bar{\epsilon})$, where $\bar{\epsilon}$ is average energy and γ is the width. The result reads

$$i\mathcal{A}(\alpha) = i\omega_+\alpha - \frac{\Gamma^2\alpha^2}{4} - \nu\int_0^\infty \frac{dx}{x}\tilde{f}_\gamma(x)e^{i(\omega-\bar{\epsilon})x}[1 - J_0(2\delta\sqrt{x\alpha})], \quad (10)$$

where $\tilde{f}_\gamma(x)$ is the Fourier transform of $f_\gamma(\epsilon)$ and J_0 is the Bessel function. Numerical results for DOS with Gaussian distribution, $\tilde{f}_\gamma(x) = e^{-\gamma x^2/2}$, are presented in Fig. 1.

Consider first the case of a strong in-plane disorder, $\Gamma/\delta \gg 1$. For a not very small γ, so that $\delta^2/\gamma\Gamma \ll 1$, the Bessel function in (10) can be expanded to first order, yielding $G(\omega) = G_W(\omega - \Sigma)$, where $G_W(\omega)$ is Wegner's Green function (that is with $\nu = 0$) and $\Sigma(\omega) = -i\nu\delta^2\int_0^\infty dx e^{-\gamma^2x^2/2+i(\omega-\bar{\epsilon})x}$ is the first-order self-energy due to the resonant scattering. If the resonant level is close to the LL center, $\omega \sim \bar{\epsilon} \ll \Gamma$, the first-order correction to the DOS reads

$$\frac{\delta g(\omega)}{g_W(0)} = -\frac{\pi - 2}{\sqrt{2}}\,\frac{\nu\delta^2}{\gamma\Gamma}\,\exp\left[-\frac{(\omega - \bar{\epsilon})^2}{2\gamma^2}\right], \quad (11)$$

where $g_W(\omega)$ is Wegner's DOS.

Resonant scattering in this case manifests itself as a minimum of width γ on top of the wider peak of width Γ. The evolution of the DOS with increasing δ/γ is shown in Fig. 1(a). The effect is strongest for $\delta/\gamma \gg 1$; however, splitting remains considerable even for $\gamma/\delta \simeq 1$. For $\delta/\gamma \ll 1$ the DOS is basically unaffected by resonant scattering and reduces to Wegner's form $g_W(\omega)$.

With increasing scattering δ/Γ, the shape of the DOS undergoes a drastic transformation [see Fig. 1(b)]. For a strong scattering, the DOS develops a *gap* in the energy interval $\omega(\omega - \bar{\epsilon}) < 0$. The existence of the gap can be traced directly to Eq. (8) (with vanishing γ/δ and Γ/δ). In this energy interval the integration path in the α integral in (9) can be rotated by $e^{-i\pi\,\mathrm{sgn}(\omega-\bar{\epsilon})/2}$,

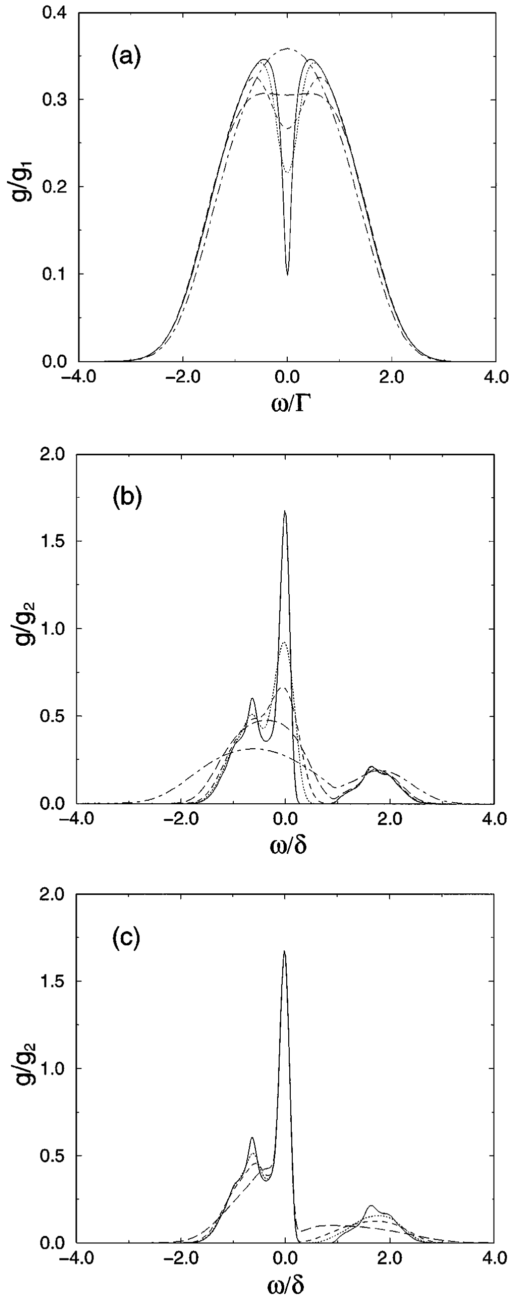

FIG. 1. (a) DOS [in units of $g_1 = (2\pi l^2)^{-1}\Gamma^{-1}$] for strong in-plane disorder, $\delta/\Gamma = 0.3$, with $\bar{\epsilon} = 0$ and $\nu = 1.5$, is shown for different $\gamma/\delta = 0.1$ (solid line), 0.5 (dotted line), 1.0 (dashed line), 2.0 (long-dashed line), and 10.0 (dot-dashed line). (b) DOS [in units of $g_2 = (2\pi l^2)^{-1}\delta^{-1}$] for strong tunneling, $\delta/\gamma = 10.0$, with $\bar{\epsilon} = \delta$ and $\nu = 0.8$, is shown for $\Gamma/\delta = 0.1$ (solid line), 0.2 (dotted line), 0.3 (dashed line), 0.5 (long-dashed line), and 1.0 (dot-dashed line). (c) DOS for weak in-plane disorder, $\Gamma/\delta = 0.1$, with $\bar{\epsilon} = \delta$ and $\nu = 0.8$, is shown for $\gamma/\Gamma = 1.0$ (solid line), 3.0 (dotted line), 5.0 (dashed line), and 10.0 (long-dashed line).

resulting in a purely real $i\mathcal{A}$. The origin of the gap is the following. If the regular disorder is weak (small Γ), the LL broadening comes from the resonant scattering alone. Then the scattering potential (3) appears to be attractive for $\omega < \bar{\epsilon}$, pulling the electronic states from the LL center to the *left*, while for $\omega > \bar{\epsilon}$ the potential is repulsive, pushing the states to the *right*. Note that for a low density of scatterers, $\nu < 1$, a fraction $1 - \nu$ of states in the plane remains unaffected. Such "condensation of states" was known also for the case of repulsive pointlike scatterers with a constant scattering strength [6,7,10,11]. In fact, the analogy extends also to the intricate structure of the DOS away from the gap. In particular, the smaller peaks correspond to singularities in $g(\omega)$ at integer values of $\omega(\omega - \bar{\epsilon})/\delta^2$ [10]. The behavior of $g(\omega)$ near the gap edges is different for $\omega \to 0$ and $\omega \to \bar{\epsilon}$: One can show that in the former case the DOS exhibits a discontinuity, $g(\omega) \propto (1 - \nu)\delta(\omega) + \mathrm{const}/|\omega|^\nu$, while near the resonance it vanishes as $(\omega - \bar{\epsilon})^{1-\nu}$. With increasing γ, the gap and the smaller peaks are washed out; however, the peak at $\omega = 0$ persists throughout [see Fig. 1(c)].

In conclusion, although our derivation was restricted to the lowest LL, we believe that our results are more general and valid for higher LL also. Indeed, the gap in the DOS for small disorder is apparently a result of the LL degeneracy. Therefore, the above argument, related to the change in the sign of the potential (3), should hold for arbitrary LL. Note that the condensation of states also occurs for all LL numbers [11]. Thus, we expect that nonanalytical ω dependence of the DOS will persist, although the precise behavior of $g(\omega)$ near the gap edges could be different. Concerning the sharp minimum in the DOS in the absence of the LL degeneracy [see Fig. 1(a)], it seems that this is a rather general feature. In fact, in the absence of magnetic field, analogous behavior has been known in the 3D case for identical scatterers [18,19].

A possible experimental realization of the multiple resonant scattering could be a system of self-assembled quantum dots separated from a 2D electron gas by a tunable tunneling barrier [4]. Because of the narrow distribution of the dots' sizes, the spread in their energy levels, γ, does not exceed 10 meV [3]. For a considerable effect of the resonant scattering, one must have $\delta^2/\gamma\Gamma \sim 1$. For a typical LL width, $\Gamma \sim 1$ meV, this condition implies that the parameter δ should be about several meV, which would be reasonable to achieve. Moreover, for $\delta/\Gamma \gtrsim 1$, an even weaker condition, the tunneling would be relatively strong and the effect of the resonant scattering would be significant. It was observed in Ref. [4] that the mobility of the 2D gas (at zero magnetic field) dropped by 2 orders of magnitude when the thickness of the tunneling barrier between the dots and the plane was reduced. Although we cannot give a quantitative estimate for the zero-field case, this is certainly in qualitative agreement with our results. We hope that our results will further motivate experiments in magnetic fields.

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