

## Coulomb drag in mesoscopic rings

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We develop a Luttinger-liquid theory of the Coulomb drag of persistent currents flowing in concentric mesoscopic rings by incorporating nonlinear corrections to the electron dispersion relation. We demonstrate that at low temperatures, interactions between electrons in different rings generate an additional phase and thus alter the *period* of Aharonov-Bohm oscillations. The resulting *nondissipative* drag depends strongly on the *relative parity* of the electron numbers. We also show that interactions set a new temperature scale below which the linear response theory does not apply at certain values of external flux. [S0163-1829(97)00319-6]

During the past decade, persistent currents (PC's) in mesoscopic rings have attracted significant interest both theoretically<sup>1-6</sup> and experimentally.<sup>7-9</sup> Much of this attention was due to a large discrepancy between experimentally observed current amplitudes in disordered metallic rings<sup>7,8</sup> and theoretical predictions based on a single-particle picture.<sup>2</sup> Yet unresolved, this puzzle has generated a number of theoretical works<sup>3</sup> on the role of electron-electron interactions in multichannel disordered rings.

At the same time, in clean single-channel rings the theory predicts that PC's at low temperatures should not be affected by interactions,<sup>10,4,6</sup> and exhibit Aharonov-Bohm oscillations as a function of flux with the same period and amplitude as for noninteracting electrons. Results of a recent experiment on a single semiconductor ring with a low number of channels seem to agree with these predictions.<sup>9</sup>

On the other hand, interactions should become essential in a system consisting of a pair of clean one-dimensional (1D) rings with different radii, placed concentrically, as is shown in Fig. 1. If the rings were isolated, PC's in each ring would oscillate with a period determined by its radius. Inter-ring interactions will change the oscillation pattern by causing a *Coulomb drag* of PC's, as we show below.

There are, in general, two physical mechanisms for the current drag. The first mechanism<sup>11</sup> originates from "friction" between two subsystems caused by the scattering of carriers in one subsystem by density fluctuations in the other. At low temperatures, the transresistance behaves as  $T^2$  ( $T^2 \ln T$  in a disordered system) and vanishes then with decreasing  $T$  as the phase space for scattering shrinks. This mechanism has been widely studied during recent years,<sup>12</sup> following experimental observation of the Coulomb drag.<sup>13</sup>

A different mechanism was pointed out by Rojo and Mahan,<sup>14</sup> who have shown that the van der Waals interaction between current carrying subsystems is modified. This leads to a *nondissipative* current drag, which remains finite as temperature goes to zero. Experimental observation of the nondissipative drag between superconductor and normal metal films<sup>15</sup> has been reported in Ref. 16.

The nondissipative mechanism is, in fact, responsible for the PC drag in mesoscopic rings. Inter-ring interactions

change the ground-state energy of the system. As a result, PC in each ring, being an *equilibrium* quantity, acquires a dependence on the flux penetrating the other ring. With decreasing temperature, this dependence should become sharper because of a singular ("sawtooth") shape of the PC as a function of flux at zero temperature. In the vicinity of a "tooth," even a small perturbation may affect strongly the *amplitude* of the PC. For this reason, an adequate theory of the PC drag must be valid beyond linear response.

In this paper we develop such a theory using the Luttinger-liquid (LL) approach to PC in 1D rings. This approach is based on Haldane's concept of topological excitations in finite-size 1D systems,<sup>17</sup> and was recently extended by Loss<sup>4</sup> to account for parity effects<sup>10</sup> in the presence of external flux. However, in its standard formulation, the LL approach does not allow for current drag, which arises from the asymmetry between electrons and holes. With electron dispersion linearized, the asymmetry is lost and the electron and hole drags completely compensate each other. In the LL formalism, the electron-hole symmetry manifests itself in a complete separation between zero modes, which carry PC, and bosonic fields, which are responsible for inter-ring interactions. We derive an interaction-induced correction to the zero mode spectrum by incorporating the lowest order nonlinear correction to the electron dispersion relation. Further, we show that at low temperatures, the entire effect of inter-ring interactions is to generate a phase that, in turn, leads to PC drag by changing the flux seen by the electrons.

We begin by recalling the LL approach<sup>17,4</sup> to interacting (spinless) electrons in a 1D ring with circumference  $L_i$

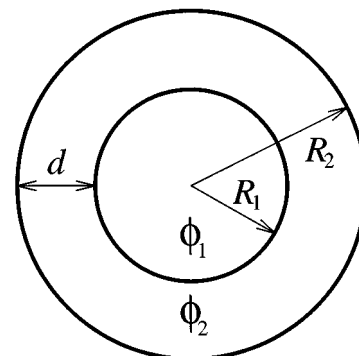


FIG. 1. Schematic picture of two concentrically placed rings.

( $i=1,2$ ) threaded by magnetic flux  $\phi_i$ . By linearizing the spectrum near the Fermi points, the electrons are decomposed into right and left moving fermions with  $\psi_{i\alpha}(x)$  ( $\alpha=\pm$ ) satisfying twisted boundary conditions  $\psi_{i\alpha}(x+L_i)=(-1)^{N_i}e^{2\pi\phi_i/\phi_0}\psi_{i\alpha}(x)$ , where the sign factor reflects the dependence of the ground state on the parity of the particle number<sup>4,10</sup>  $N_i$ , and  $\phi_0$  is the flux quantum. The fermion operator can be expressed through bosonic fields as  $\psi_{i\alpha}(x)=(2\epsilon L_i)^{-1/2}e^{-i\varphi_{i\alpha}(x)}$ , with  $\varphi_{i\alpha}(x)=[\theta_i(x)-\alpha\varphi_i(x)]/2$ . The fields  $\varphi_i(x)$  and  $\theta_i(x)$  can be presented as<sup>18</sup>

$$\varphi_i(x)=\varphi_{iJ}+M_i2\pi x/L_i+\bar{\varphi}_i(x), \quad (1)$$

$$\theta_i(x)=\theta_{iM}+(J_i-2\Phi_i)2\pi(x+L_i/2)/L_i+\bar{\theta}_i(x), \quad (2)$$

where  $M_i$  and  $J_i$  are fermion number and (topological) current operators, and  $\theta_{iM}$  and  $\varphi_{iJ}$  are their conjugates:  $[\theta_{iM},M_i]=[\varphi_{iJ},J_i]=2i\delta_{ii'}$ . For further convenience, we have incorporated both the flux and parity dependence into a single quantity  $\Phi_i$ , which is defined as  $\Phi_i=\phi_i/\phi_0$  for  $N_i$  odd, and  $\Phi_i=\phi_i/\phi_0-1/2$  for  $N_i$  even. Owing to the boundary conditions, the eigenvalues of  $J_i$  and  $M_i$  satisfy the selection rule that their sum is even. The periodic fields  $\bar{\varphi}_i(x)$  and  $\bar{\theta}_i(x)$  have the usual representation in terms of bosonic creation and annihilation operators

$$\bar{\varphi}_i(x)=\sum_{k_i\neq 0}\left|\frac{2\pi}{L_ik_i}\right|^{1/2}e^{ik_ix}(a_{ik_i}^\dagger+a_{ik_i}), \quad (3)$$

$$\bar{\theta}_i(x)=\sum_{k_i\neq 0}\left|\frac{2\pi}{L_ik_i}\right|^{1/2}\text{sgn}(k_i)e^{ik_ix}(a_{ik_i}^\dagger-a_{ik_i}), \quad (4)$$

where  $k_i=2\pi p/L_i$  ( $p$  being an integer), and the regularization factor  $\exp(-\epsilon|k_i|L_i/2\pi)$  is implicit. In the absence of backscattering,  $\varphi_i$  is related to the charge density (relative to the background density  $N_i/L_i$ ) by  $\rho_i=\partial_x\varphi_i/2\pi$ . The eigenvalues of  $M_i$  represent the numbers of extra electrons added to the ring, while those of  $J_i$  are the excess of left over right moving fermions.<sup>17</sup>

In terms of the bosonic fields  $\varphi_i$ , the (normal ordered) Hamiltonian of free fermions with linear dispersion reads

$$H_0=\sum_i\frac{v_i}{8\pi}\int_0^{L_i}dx:(4\pi\Pi_i)^2+(\partial_x\varphi_i)^2:, \quad (5)$$

where  $v_i$  is the Fermi velocity and  $\Pi_i\equiv P_i-\Phi_i/L_i=\partial_x\theta_i/4\pi$ ,  $P_i$  being the canonical momentum.

In calculating PC's it is convenient to use the Lagrangian formulation and present the partition function  $Z$  in terms of a functional integral. The free (Euclidean) action has the form

$$S_0=\sum_i\int_0^\beta d\tau\int_0^{L_i}dx\left\{\frac{1}{8\pi v_i}[(\partial_\tau\varphi_i)^2+v_i^2(\partial_x\varphi_i)^2]-\frac{i}{L_i}(\Phi_i+\delta_i)\partial_\tau\varphi_i\right\}, \quad (6)$$

where  $\beta$  is the inverse temperature and  $\varphi(\tau,x)$  has a decomposition (up to a constant)

$$\varphi_i(\tau,x)=2\pi n_i\tau/\beta+2\pi m_ix/L_i+\bar{\varphi}_i(\tau,x). \quad (7)$$

Here zero modes  $n_i$  and  $m_i$  are winding numbers in  $\tau$  and  $x$  directions, respectively, and  $\delta_i$  in Eq. (6) enforces the selection rule by taking values  $\delta_i=0(1/2)$  for  $m_i$  even (odd).<sup>4</sup> Note that  $m_i$  coincides with the eigenvalue of  $M_i$ . In the following we assume the number of electrons in the rings fixed and restrict ourselves to the  $m_i=0$  (with  $\delta_i=0$ ) sector.

The partition function is given by  $Z=\int D\varphi e^{-S}$ , where  $S=S_0+S_{int}$ . Here the term

$$S_{int}=\frac{1}{8\pi^2}\sum_{ij}\int_0^\beta d\tau\int_0^{L_i}dx\int_0^{L_j}dx'\partial_x\varphi_i V_{ij}(x,x')\partial_{x'}\varphi_j, \quad (8)$$

with  $V_{ij}$  being the Coulomb potential, describes intraring and inter-ring interactions, and the measure  $D\varphi$  includes the sum over zero modes together with the integral over periodic fields  $\bar{\varphi}_i$ . The PC is found by separating out the contribution from the zero modes. With  $\varphi_i(\tau,x)$  of the form (7) (with  $m_i=0$ ), the latter are completely decoupled and the action takes the form

$$S=\sum_i\left(\frac{n_i^2}{\beta T_i}-2\pi in_i\Phi_i\right)+\bar{S}, \quad (9)$$

where  $\bar{S}=S_0[\bar{\varphi}]+S_{int}[\bar{\varphi}]$  is flux independent and  $T_i=2v_i/\pi L_i$  is the temperature scale set by the finite size of each ring. Thus, the PC,  $\mathcal{J}_i=\beta^{-1}\partial_{\phi_i}\ln Z$ , is carried by zero modes whose spectrum is *unaltered* by inter-ring interactions. This is a consequence of the linearization of the electron dispersion and the resulting quadratic form of the bosonic action  $S_0$ . Physically, linearization lifts the electron-hole asymmetry, which is responsible for the current drag, so that PC in each ring is sustained by its own flux  $\phi_i$ .

Let us now consider the lowest nonlinear correction to the fermion dispersion relation. Following Ref. 17, we take the correction to the linearized fermion Hamiltonian as

$$H_{nl}=-\frac{1}{2m_e}\sum_{i\alpha}\int_0^{L_i}dx:\psi_{i\alpha}^\dagger(x)\partial_x^2\psi_{i\alpha}(x):, \quad (10)$$

where  $m_e$  is electron mass. Transforming Eq. (10) with the help of bosonic representation of  $\psi_{i\alpha}(x)$ , we obtain

$$H_{nl}=\frac{1}{48\pi m_e}\sum_i\int_0^{L_i}dx:(\partial_x\varphi_i)^3+3\partial_x\varphi_i(4\pi\Pi_i)^2:, \quad (11)$$

which means that the corresponding bosonic theory is no longer free. Regarding  $H_{nl}$  as a small term, we can obtain a correction to the action by adding Eq. (11) to Eq. (5) and repeating the steps leading to the functional-integral representation of  $Z$ . To the first order in  $1/m$ , the correction takes the form

$$S_{nl}=\frac{1}{48\pi m_e}\sum_i\int_0^\beta d\tau\int_0^{L_i}dx\left[(\partial_x\varphi_i)^3-\frac{3}{v_i^2}\partial_x\varphi_i(\partial_\tau\varphi_i)^2\right]. \quad (12)$$

Using Eq. (7) (with  $m_i=0$ ), we separate out the zero-mode contribution in  $S_{nl}$  by writing

$$S_{nl} = \sum_i \frac{n_i}{4\beta m_e v_i^2} \int_0^\beta d\tau \int_0^{L_i} dx \bar{\varphi}_i \partial_\tau \partial_x \bar{\varphi}_i + \bar{S}_{nl}, \quad (13)$$

where  $\bar{S}_{nl}$  depends only on fields  $\bar{\varphi}_i$ , which are now coupled to the zero modes via the first term. Adding Eq. (13) to Eq. (9), we then perform the functional integral  $e^{-\beta\bar{F}} = \int D\bar{\varphi} e^{-\bar{S} - S_{nl}}$ . Note that after the zero modes are separated out, the small term  $\bar{S}_{nl}$  can be neglected and the remaining Gaussian integral is explicitly evaluated. The resulting  $\bar{F}(n_1, n_2)$  admits an expansion in terms of  $1/m_e$  with the odd orders vanishing due to translational invariance. Thus, to the first nonvanishing order

$$\bar{F}(n_1, n_2) = \bar{F}_0 + \sum_{ij} \bar{F}_{ij} n_i n_j, \quad (14)$$

where  $\bar{F}_0$  is  $n_i$  and  $\phi_i$  is independent. The second term in Eq. (14), which is to be combined with the first term in Eq. (9), represents the correction sought to the zero-mode spectrum. The diagonal coefficients  $F_{ii}$  can be absorbed into  $T_i$  via renormalization of the Fermi velocities and do not play any role in the following. Factorizing out the zero-mode part of the partition function,  $Z_0 \equiv e^{-\beta F_0} = e^{\beta \bar{F}_0} Z$ , we finally arrive at

$$Z_0 = \sum_{n_1 n_2} \exp\left(-\frac{1}{\beta} \sum_{ij} a_{ij} n_i n_j + 2\pi i \sum_i n_i \Phi_i\right), \quad (15)$$

where  $a_{ii} = T_i^{-1}$ , and the parameter  $a_{12} = \beta^2 \bar{F}_{12}$ , given by

$$a_{12} = \frac{-1}{16m_e^2 v_1^2 v_2^2 \beta} \times \int dz_1 \int dz_2 \partial_{\tau_1} \partial_{x_1} \bar{D}_{12}(z_1, z_2) \partial_{\tau_2} \partial_{x_2} \bar{D}_{21}(z_2, z_1), \quad (16)$$

describes the *coupling* between zero modes in different rings. Here  $z_i = (\tau_i, x_i)$  and  $\bar{D}_{12}(z_1, z_2)$  is the nondiagonal part of the boson Green function  $\bar{D}_{ij}(z_1, z_2) = \langle \bar{\varphi}_i(z_1) \bar{\varphi}_j(z_2) \rangle$ , calculated from the action  $\bar{S}$ . Due to the azimuthal symmetry of the system, it is convenient to adopt the angular-momentum representation, in which the Fourier transform of the inverse Green function reads

$$\bar{D}_{ij}^{-1}(\omega, p) = \frac{\delta_{ij}}{4\pi v_i L_i} \left[ \omega^2 + v_i^2 \left( \frac{2\pi p}{L_i} \right)^2 \right] + \frac{p^2}{L_i L_j} V_{ij}(p), \quad (17)$$

where  $V_{ij}(p)$  is the Fourier transform of the Coulomb interaction. For  $d \ll L_i$ , it can be presented as

$$V_{ij}(p) \approx \frac{2e^2}{\kappa(L_i L_j)^{1/2}} K_0 \left[ \frac{2\pi(p+1/2)d_{ij}}{(L_i L_j)^{1/2}} \right], \quad (18)$$

where  $\kappa$  is the dielectric constant,  $K_0$  is the modified Bessel function, and we used the notation  $d_{ij} = d$  for  $i \neq j$ , and  $d_{ii} = w$ ,  $w$  being the width of the rings. The expression for  $a_{12}$  then takes the form

$$a_{12} = \frac{-\pi^2}{(2m_e v_1 v_2 L_1 L_2)^2 \beta} \sum_{\omega p} \omega^2 p^2 [\bar{D}_{12}(\omega, p)]^2. \quad (19)$$

The frequency sum in Eq. (19) is straightforward and yields

$$a_{12} = \frac{-2\pi^2}{m_e^2 v_1 v_2 L_1 L_2 (T_1 T_2)^{1/2}} \times \sum_{p>0} p v_{12} \frac{\partial}{\partial v_{12}} \left[ \left( \frac{Q_+}{t_+} - \frac{Q_-}{t_-} \right) \frac{1}{Q_+^2 - Q_-^2} \right], \quad (20)$$

where

$$t_{\pm} = \tanh \left[ \frac{\pi^2}{2} \left( \frac{T_1}{T} \frac{T_2}{T} \right)^{1/2} p Q_{\pm} \right], \quad Q_{\pm}(p) = \sqrt{A(p) \pm B(p)}, \quad (21)$$

$$A(p) = [r(1+v_{11}) + r^{-1}(1+v_{22})]/2,$$

$$B(p) = \sqrt{[r(1+v_{11}) - r^{-1}(1+v_{22})]^2/4 + v_{12}^2}, \quad (22)$$

$$v_{ij}(p) = \frac{2}{\pi^2} (T_i T_j)^{-1/2} V_{ij}(p) = \frac{2e^2}{\kappa \pi (v_i v_j)^{1/2}} K_0 \left[ \frac{2\pi(p+1/2)d_{ij}}{(L_i L_j)^{1/2}} \right], \quad (23)$$

and  $r = (T_1/T_2)^{1/2}$  characterizes the asymmetry between the rings. To estimate  $a_{12}$  we assume, for simplicity, that the rings are identical, with  $v_i = v$ ,  $T_i = T_0$ , and  $L_i = L$ . Then the analysis of Eq. (20) shows that for  $T \ll T_d \equiv T_0(L/d)\sqrt{1 + \alpha \ln(2d/w)}$ , where  $\alpha = 2e^2/\kappa \pi v$  is dimensionless interaction constant, the coupling  $a_{12}$  is *temperature independent*. Since for  $d \ll L$  one has  $T_d \gg T_0$ , and  $T_0$  is the crossover temperature above which the PC is exponentially suppressed,  $a_{12}$  can be regarded as a constant in the entire range of relevant temperatures. For  $d \gg w$  the screening is strong and  $a_{12}$  is determined by the first order in the inter-ring interaction. The right-hand side of Eq. (20) then gives

$$a_{12} \approx -\frac{\alpha^2}{32T_0 d^2 k_F^2 [1 + \alpha \ln(2d/w)]^{5/2}}, \quad (24)$$

where  $k_F$  is the Fermi momentum. A  $d^{-2}$  dependence (without logarithmic factor) was obtained previously for the infinite wires.<sup>14</sup>

Turning to the partition function (15), we observe that since the zero modes in different rings are now coupled, the PC in ring  $i$ ,  $\mathcal{J}_i = -\partial F_0/\partial \phi_i$ , depends also on the flux through ring  $j$ . The coupling  $a_{12}$  being small, one might be tempted to obtain a correction to the current by expanding the free energy to first order in  $a_{12}$ . It is important to realize, however, that at low temperatures such expansion does *not* exist for all values of  $\Phi_i$ . In order to make this point clear, we rewrite Eq. (15) in a different form using Poisson's formula (omitting constant prefactor)

$$Z_0 = \sum_{p_1 p_2} \exp \left[ -\beta \sum_{ij} (p_i - \Phi_i) c_{ij} (p_j - \Phi_j) \right], \quad (25)$$

where  $\hat{c} = \pi^2 \hat{a}^{-1}$ . For  $a_{12} \ll a_{ii}$  one has for diagonal elements  $c_{ii} = \pi^2 T_i$  which, in the absence of interactions, is the level spacing at the Fermi level. With such a form of  $Z_0$ , the PC is given by ( $i \neq j$  in the rest of the paper)

$$\mathcal{J}_i(\Phi_1, \Phi_2) = \frac{2}{\phi_0} [c_{ii}(\langle p_i \rangle - \Phi_i) + c_{12}(\langle p_j \rangle - \Phi_j)], \quad (26)$$

where  $\langle p_i \rangle$  stands for the average of  $p_i$  calculated from the partition function (25). Note that  $\Phi_i$  in Eq. (26) takes values in the interval  $0 < \Phi_i < 1$ , and  $\mathcal{J}_i$  is periodically continued outside of this interval. For further analysis, it is convenient to obtain an equation for  $\langle p_i \rangle$ . This can be done by making use of the following identity

$$\langle p_i \rangle = \frac{1}{2} - \frac{1}{2} \sum_{n=-\infty}^{\infty} \langle \tanh \beta [c_{ii}(n + 1/2 - \Phi_i) + c_{12}(p_j - \Phi_j)] \rangle, \quad (27)$$

which can be easily derived by substituting Jacoby's product identity for the  $\theta_3$  function<sup>19</sup> in place of the sum over variable  $p_i$  in the double sum (25). For low temperatures,  $T \ll c_{ii}$ , the fluctuations of  $p_i$  are suppressed and all moments factorize,  $\langle p_i^m \rangle = \langle p_i \rangle^m$ , which allows one to replace  $p_i$  by  $\langle p_i \rangle$  in the right-hand side of Eq. (27). It is also easy to see that for such temperatures, all terms in the sum with  $n \neq 0$  cancel each other out (up to exponentially small corrections). Then Eq. (27) simplifies to the system

$$\langle p_i \rangle = f_0 [c_{ii}(1 - 2\Phi_i) + 2c_{12}(\langle p_j \rangle - \Phi_j)], \quad (28)$$

where  $f_0(x) = (e^{\beta x} + 1)^{-1}$  is the Fermi function. This system, together with Eq. (26) determines the PC at low temperatures,  $T \ll c_{ii} = \pi^2 T_i$ . In the absence of inter-ring coupling ( $c_{12} = 0$ ), one recovers the PC for an isolated ring,

$$\mathcal{J}_i^0(\Phi_i) = 2I_i \{f_0[c_{ii}(1 - 2\Phi_i)] - \Phi_i\}, \quad (29)$$

where  $I_i = c_{ii}/\phi_0 = ev_i/L_i$  is the current amplitude. The current  $\mathcal{J}_i^0$  is a linear function of flux except in the interval  $\Phi_i - 1/2 \sim T/c_{ii} \ll 1$  in which it rapidly changes from  $-I_i$  to  $I_i$ . Note that  $\Phi_i$  depends on the parity of the total number of electrons,  $N_i$ , resulting in a diamagnetic (paramagnetic) current for  $N_i$  odd (even).<sup>10,4</sup>

Turning the coupling on, PC's can be found by iterating the system (28). First, in the argument of the Fermi function we substitute  $\langle p_i \rangle$  expressed via  $\mathcal{J}_i$  by neglecting the second term in Eq. (26). Substituting Eq. (28) back into Eq. (26), we finally obtain

$$\mathcal{J}_i(\Phi_1, \Phi_2) = \mathcal{J}_i^0(\Phi_i - \delta_i), \quad \delta_i = b \mathcal{J}_j(\Phi_1, \Phi_2), \quad (30)$$

with  $b = \phi_0 c_{12} / 2c_{11} c_{22} = -\phi_0 a_{12} / 2\pi^2$ . Equation (30), which is our main result, describes the mutual dependence (Coulomb drag) of PC's in coupled rings. It shows that electrons encircling the ring  $i$  acquire additional (Berry's) phase proportional to PC in the ring  $j$ . Thus, *inter-ring interactions change the period of the Aharonov-Bohm oscillations*. In particular, peak positions of PC get shifted by an amount  $\delta_i$ . Remarkably, the resulting oscillation pattern depends strongly on the *relative parity* of electron numbers  $N_i$ , as it can be seen in Fig. 2. We emphasize that in the vicinity of

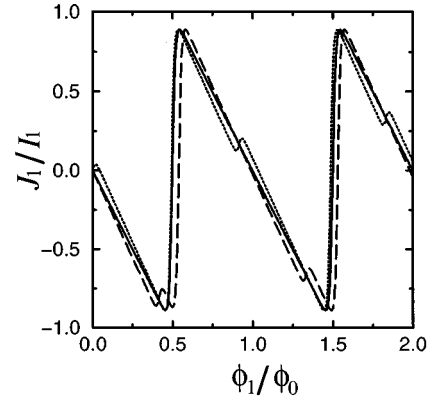


FIG. 2. Example of PC in ring 1 at  $\phi_2/\phi_1 = 1.1$  and  $T_i/T = 5.0$  for different inter-ring couplings:  $c_{12}/c_{ii} = 0$  (solid line), and  $c_{12}/c_{ii} = 0.1$  with the same (dashed line) and opposite (dotted line) parities of electron numbers.

the peak,  $\Phi_i \approx 1/2$ , PC (30) *cannot* be expanded in  $\delta_i$  if the temperature is low enough:  $T \lesssim T^* \equiv c_{12}$ . Similarly,  $\mathcal{J}_j(\Phi_1, \Phi_2)$  in the expression for  $\delta_i$  cannot be replaced by  $\mathcal{J}_j^0(\Phi_j)$  for  $\Phi_j \approx 1/2$ . If  $\Phi_i$  is not too close to  $1/2$ , one can neglect  $\delta_i$  in the argument of the Fermi function and the correction to PC takes the form  $\delta \mathcal{J}_i / I_i = 2b \mathcal{J}_j$  (note that  $b$  is positive).

Equation (30) was obtained for temperatures lower than  $T_i$ . With increasing  $T$ , the PC amplitude gets damped due to the loss of phase coherence. Expanding the exponent in Eq. (25) in terms of  $c_{12}$ , the correction to the free energy can be written as

$$\delta F_0 = \phi_0 b \mathcal{J}_1^0(\Phi_1) \mathcal{J}_2^0(\Phi_2), \quad (31)$$

with  $\mathcal{J}_i^0(\Phi_i) = (4I_i/\pi)(T/T_i)e^{-T/T_i} \sin(2\pi\Phi_i)$ . Correspondingly, the correction to PC for  $T \gg T_i$  is suppressed by an additional damping factor.

Let us now address the experimental implications of our result. PC's with amplitude of about 4 nA were observed<sup>9</sup> in a micrometer-size  $\text{Ga}_{1-x}\text{Al}_x\text{As}/\text{GaAs}$  ring with width  $w \approx 160$  nm, and Fermi wavelength of electrons  $\lambda_F \approx 40$  nm, which corresponds to  $\alpha \approx 0.5$  and  $T_0 \approx 200$  mK. Assuming that two concentric rings were fabricated on the same plane, the estimate of  $a_{12}$  for the above parameters and  $d \sim w$  yields a small phase shift  $\delta \sim 10^{-4}$ . However, this small value is due to the rather large distance between rings in the plane. The situation will be significantly improved if two concentric rings were fabricated on parallel 2D layers. Since the inter-layer distance can be as small as 3 nm,<sup>20</sup> this would allow one to decrease the actual separation between current carrying channels by 1–2 orders of magnitude. A reliable estimate of  $a_{12}$  for  $d \ll w$  would require careful treatment of the screened interaction between the two closely spaced rings and depend on details of the structure. The  $d$  dependence (24) suggests, however, that in this case one might expect a much larger phase shift, which would be within resolution of experimental observations.

Finally, we note that the role of Coulomb interactions in the mutual dependence of PC's appears, in fact, somewhat analogous to that of magnetic inductance of two current loops. Indeed, the latter shifts the flux from  $\Phi_i$  to

$\Phi_i + b_m \mathcal{J}_j$ , where  $b_m$  is the coefficient of mutual inductance. This merely replaces  $b$  in Eq. (30) by  $b - b_m$ , leaving the rest of our discussion intact. Thus, the *sign* of the shift would indicate which mechanism, in fact, dominates for a particular experimental realization. The estimate of the relative strength  $b/b_m$  can be obtained for the case  $w \lesssim d \ll L$  with<sup>21</sup>  $b_m \approx (2Le/c^2) \ln(L/d)$ , yielding  $b/b_m \approx (c/v)^2 \delta / \pi \alpha \kappa \ln(L/d)$

(restoring the speed of light  $c$ ). For the above parameters (with  $\delta \sim 10^{-4}$ ) this ratio is of order unity. For the double-layer geometry with significantly larger  $\delta$ , we expect that Coulomb interactions will dominate.

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