1. A rigid body consists of three thin uniform rods, each of mass $m$ and length $2a$, held mutually perpendicular at their midpoints. In the co-ordinate system where the origin is at the point of intersection and the axes are along the rods:

(a) Find the angular momentum and kinetic energy of the body if it rotates with angular velocity $\omega$ about an axis passing through the origin and the point $(1, 1, 1)$.

(b) Show that the moment of inertia is the same for rotations about any axis passing through the origin.

2. Consider a solid uniform cone of mass $m$, base $2a$, and height $h$. Choose the origin of your co-ordinate system to be at the apex of the cone, and the $z$-axis to lie along the symmetry axis of the cone.

(a) Compute the inertia tensor for the cone in this co-ordinate system;

(b) What are the principal axes?

3. A three-particle system consists of masses $m_i$ with co-ordinates $\mathbf{r}_i$ as follows:

\[
\begin{align*}
& m_1 = m; & & \mathbf{r}_1 = (b, 0, b); \\
& m_2 = 3m; & & \mathbf{r}_2 = (0, 0, -b); \\
& m_3 = 2m; & & \mathbf{r}_3 = (-b, 0, b).
\end{align*}
\]

Compute the inertia tensor for this system, the principal axes, and the principal moments of inertia of this system for rotations about the origin.
4. A thin, uniform, rectangular plate of mass $m$ has dimensions $2a$ by $a$. Choose a coordinate system such that the plate lies in the $xy$-plane with the long edge along the $x$ axis and the short edge along the $y$ axis. (See figure below.)

(a) Compute the inertia tensor for this system;
(b) How should the coordinate system be rotated so that the axes lie along the principal axes of the plate?
(c) Hence, or otherwise, compute the angular momentum and rotational kinetic energy of the plate if it is spinning with angular speed $\omega$ about the diagonal passing through the origin.